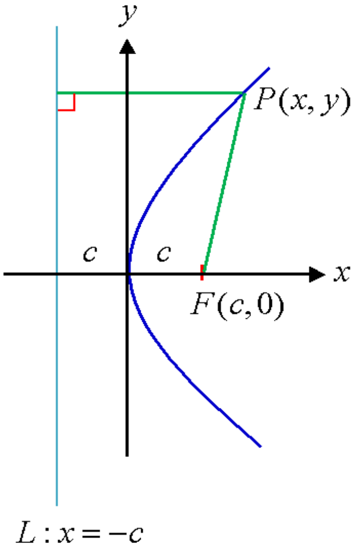
【拋物線標準式】



$\overline{PF} = d(P, L)$

$\sqrt{(x-c)^2 + y^2} = x + c$

$(x-c)^2 + y^2 = (x+c)^2$

$y^2 = (x+c)^2 - (x-c)^2$

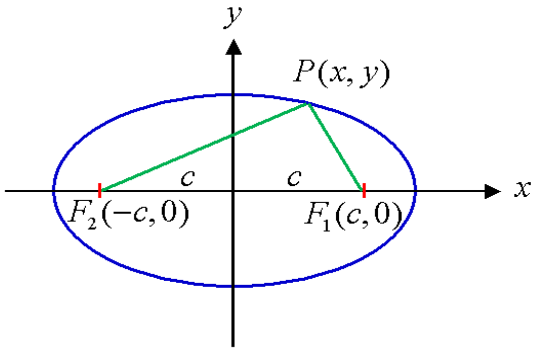
$y^2 = [(x+c)-(x-c)][(x+c)+(x-c)]$

$y^2 = (2c)\cdot(2x)$

$y^2 = 4cx$

※ $4c$ 為正焦弦長

【橢圓標準式】



$$\overline{PF_1}+\overline{PF_2}=2a$$

$$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$$

$$\sqrt{(x+c)^2+y^2}=2a-\sqrt{(x-c)^2+y^2}$$

$$[\sqrt{(x+c)^2+y^2}]^2=[2a-\sqrt{(x-c)^2+y^2}]^2$$

$$(x+c)^2+\cancel{y^2}=4a^2-4a\sqrt{(x-c)^2+y^2}+(x-c)^2+\cancel{y^2}$$

$$(x+c)^2-(x-c)^2=4a^2-4a\sqrt{(x-c)^2+y^2}$$

$$\cancel{4}cx=\cancel{4}a^2-\cancel{4}a\sqrt{(x-c)^2+y^2}$$

$$a\sqrt{(x-c)^2+y^2}=a^2-cx$$

$$[a\sqrt{(x-c)^2+y^2}]^2=(a^2-cx)^2$$

$$a^2(x^2-2cx+c^2+y^2)=a^4-2a^2cx+c^2x^2$$

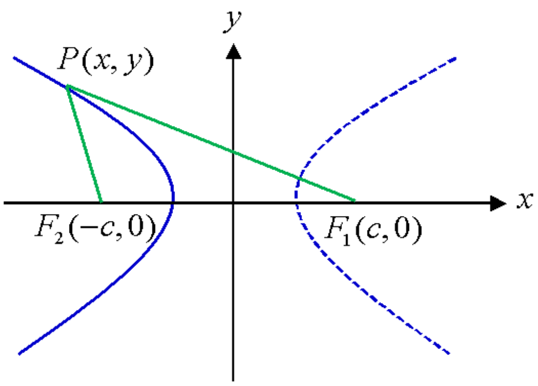
$$a^2x^2-\cancel{2a^2cx}+a^2c^2+a^2y^2=a^4-\cancel{2a^2cx}+c^2x^2$$

$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

令 $a^2-c^2=b^2$

則 $b^2x^2+a^2y^2=a^2b^2 \Rightarrow \boxed{\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1}$

【雙曲線標準式】



① $\overline{PF_1}-\overline{PF_2}=2a$

$\sqrt{(x-c)^2+y^2}-\sqrt{(x+c)^2+y^2}=2a$

$\sqrt{(x-c)^2+y^2}=2a+\sqrt{(x+c)^2+y^2}$

$[\sqrt{(x-c)^2+y^2}]^2=[2a+\sqrt{(x+c)^2+y^2}]^2$

$(x-c)^2+y^2=4a^2+4a\sqrt{(x+c)^2+y^2}+(x+c)^2+y^2$

$(x-c)^2-(x+c)^2=4a^2+4a\sqrt{(x+c)^2+y^2}$

$-4cx=4a^2+4a\sqrt{(x+c)^2+y^2}$

$-a\sqrt{(x+c)^2+y^2}=a^2+cx$

$[-a\sqrt{(x+c)^2+y^2}]^2=(a^2+cx)^2$

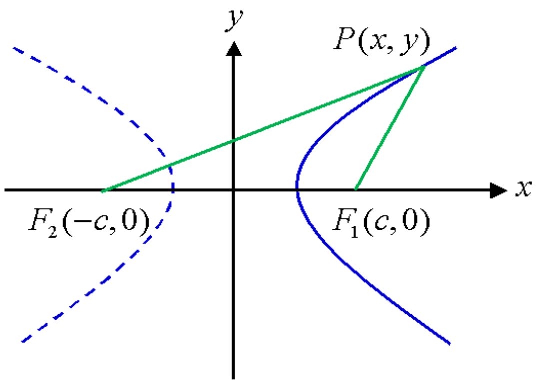
$a^2(x^2+2cx+c^2+y^2)=a^4+2a^2cx+c^2x^2$

$a^2x^2+2a^2cx+a^2c^2+a^2y^2=a^4+2a^2cx+c^2x^2$

$(c^2-a^2)x^2-a^2y^2=a^2(c^2-a^2)$

令 $c^2-a^2=b^2$

則 $b^2x^2-a^2y^2=a^2b^2 \Rightarrow \dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ (雙曲線的左支線)



② $\overline{PF_2}-\overline{PF_1}=2a$

$\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=2a$

$\sqrt{(x+c)^2+y^2}=2a+\sqrt{(x-c)^2+y^2}$

$[\sqrt{(x+c)^2+y^2}]^2=[2a+\sqrt{(x-c)^2+y^2}]^2$

$(x+c)^2+y^2=4a^2+4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2$

$(x+c)^2-(x-c)^2=4a^2+4a\sqrt{(x-c)^2+y^2}$

$4cx=4a^2+4a\sqrt{(x-c)^2+y^2}$

$-a\sqrt{(x-c)^2+y^2}=a^2-cx$

$[-a\sqrt{(x-c)^2+y^2}]^2=(a^2-cx)^2$

$a^2(x^2-2cx+c^2+y^2)=a^4-2a^2cx+c^2x^2$

$a^2x^2-2a^2cx+a^2c^2+a^2y^2=a^4-2a^2cx+c^2x^2$

$(c^2-a^2)x^2-a^2y^2=a^2(c^2-a^2)$

令 $c^2-a^2=b^2$

則 $b^2x^2-a^2y^2=a^2b^2 \Rightarrow \dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ (雙曲線的右支線)