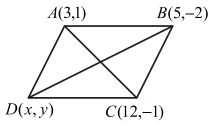
九十八學年四技二專第五次聯合模擬考試
共同考科　數學(C)卷　詳解

數學(C)卷 98-5-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	A	B	B	C	C	D	A	A	D	A	C	D	A	B	D	C	C	C	C	D	A	B	A	B

1. 設 D 點坐標爲 (x,y)



∵平行四邊形對角線互相平分

$\therefore \frac{A+C}{2}=\frac{B+D}{2}\Rightarrow A+C=B+D$

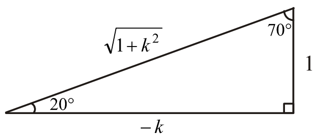
$\Rightarrow \begin{cases}3+12=5+x\\1+(-1)=(-2)+y\end{cases}\Rightarrow\begin{cases}x=10\\y=2\end{cases}$，$\therefore D(10,2)$

2. $\cot 200^\circ=\cot(180^\circ+20^\circ)=\cot 20^\circ=-k$

$\therefore \sin 1330^\circ=\sin(180^\circ\times 7+70^\circ)$

$=-\sin 70^\circ=-\frac{-k}{\sqrt{1+k^2}}$

$=\frac{k}{\sqrt{1+k^2}}$



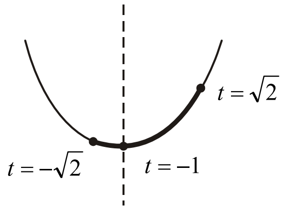
3. 令 $t=\sin x+\cos x$

$\Rightarrow -\sqrt{2}\le t\le\sqrt{2}$

故 $f(x)=(\sin x+\cos x)^2$

$+2(\sin x+\cos x)$ 可寫爲

$f(t)=t^2+2t=(t+1)^2-1$



$\therefore t=\sqrt{2}$ 時

$f(t)$ 有最大值 $M=(\sqrt{2})^2+2\sqrt{2}=2+2\sqrt{2}$

$t=-1$時，$f(t)$ 有最小值 $m=-1$

4. $2\sin\theta=3\cos\theta\Rightarrow 4\sin^2\theta=9\cos^2\theta$

$\because \sin^2\theta+\cos^2\theta=1$，$\therefore 4\sin^2\theta=9(1-\sin^2\theta)$

$\Rightarrow 13\sin^2\theta=9\Rightarrow\sin^2\theta=\frac{9}{13}\Rightarrow\sin\theta=\pm\frac{3}{\sqrt{13}}$

(1) $\sin\theta=\pm\frac{3}{\sqrt{13}}\Rightarrow\cos\theta=\pm\frac{2}{\sqrt{13}}$

$\because 2\sin\theta=3\cos\theta\ \therefore\sin\theta$ 和 $\cos\theta$ 同號 $\Rightarrow\sin 2\theta>0$

故 $\sin 2\theta=2\sin\theta\cos\theta=2\times\frac{3}{\sqrt{13}}\times\frac{2}{\sqrt{13}}=\frac{12}{13}$

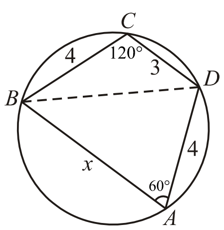
(2) $\cos 2\theta=1-2\sin^2\theta=1-2\times\frac{9}{13}=-\frac{5}{13}$

5. 設 $\overline{AB}=x$，作 $\overline{BD}$

∵圓內接四邊形對角互補

$\therefore \angle BAD=60^\circ$

根據餘弦定理



ΔABD 中的 $\overline{BD}$

$=\Delta BCD$ 中的 $\overline{BD}$

$\Rightarrow \overline{BD}^2=x^2+4^2-2\times x\times 4\cos 60^\circ$

$=4^2+3^2-2\times 4\times 3\cos 120^\circ$

$\Rightarrow x^2-4x-21=0\Rightarrow(x-7)(x+3)=0$

$\Rightarrow x=7$ 或 -3（-3 不合）$\therefore \overline{AB}=7$

6. $|2\vec{a}+3\vec{b}|^2=4|\vec{a}|^2+12\,\vec{a}\cdot\vec{b}+9|\vec{b}|^2$

$=4|\vec{a}|^2+12\times|\vec{a}||\vec{b}|\cos\frac{\pi}{3}+9|\vec{b}|^2$

$=4\times 3^2+12\times 3\times 2\times\frac{1}{2}+9\times 2^2=108$

$\therefore |2\vec{a}+3\vec{b}|=\sqrt{108}=6\sqrt{3}$

7. 利用綜合除法

如右圖所示

∴商式 $Q(x)$

$=x^3+3x^2-4x+2$

餘式 $R(x)=-3$

```
    2  +5 −11  +8  −5 | 1/2
       +1  +3  −4  +2 |
2 | 2  +6  −8  +4 | −3
    1  +3  −4  +2
```

8. 令 $x=2+\sqrt{7}i$

$\Rightarrow x-2=\sqrt{7}i$

$\Rightarrow x^2-4x+4=-7$

$\Rightarrow x^2-4x+11=0$

爲以 $2+\sqrt{7}i$

爲一根之二次方程式

```
                1 − 3
1 − 4 + 11 ) 1 − 7       + a          + b
             1 − 4       + 11
                 − 3     + (a − 11)   + b
                 − 3     + 12         − 33
                           (a − 23) + (b + 33)
```

$\Rightarrow x^2-4x+11$ 可整除 x^3-7x^2+ax+b

$\Rightarrow\begin{cases}a-23=0\Rightarrow a=23\\b+33=0\Rightarrow b=-33\end{cases}$

又 $x^3-7x^2+ax+b=(x^2-4x+11)(x-3)$

∴第三個根爲 $c=3$，故 $a+b+c=23-33+3=-7$

9. $\because |z_1|=\sqrt{2}|z_2|\ \therefore\frac{|z_1|}{|z_2|}=\sqrt{2}$ 又 $\frac{z_1}{z_2}$ 的主幅角爲 $\frac{3\pi}{4}$

$\Rightarrow\frac{z_1}{z_2}=\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})=\sqrt{2}(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i)$

$=-1+i\Rightarrow\frac{3-4i}{z_2}=-1+i$

$\Rightarrow z_2=\frac{3-4i}{-1+i}=\frac{(3-4i)(-1-i)}{(-1+i)(-1-i)}=\frac{-7}{2}+\frac{1}{2}i$

10. $(\sqrt{27})^{4(x+2)}<(\frac{1}{9})^{-x^2-2}\Rightarrow(3^{\frac{3}{2}})^{4(x+2)}<(3^{-2})^{-x^2-2}$

$\Rightarrow\frac{3}{2}\times 4(x+2)<-2(-x^2-2)\Rightarrow x^2-3x-4>0$

$\Rightarrow(x-4)(x+1)>0\Rightarrow x>4$ 或 $x<-1$

11. $\log_2(x-1)=1+\log_4(x+2)$

$\Rightarrow \log_4(x-1)^2 = \log_4 4 + \log_4(x+2) = \log_4 4(x+2)$

$\Rightarrow (x-1)^2 = 4(x+2) \Rightarrow x^2 - 6x - 7 = 0$

$\Rightarrow (x-7)(x+1) = 0 \Rightarrow x = 7$ 或 -1

(∵ -1 會使眞數 $x-1<0$ ∴ -1 不合)

12. $a_1 + a_2 + \cdots + a_{21} = 0 \Rightarrow \frac{21}{2}(2a_1 + 20d) = 0$

$\Rightarrow 2a_1 + 20d = 0 \Rightarrow a_1 + 10d = 0 \Rightarrow a_1 = -10d$

又 $a_7 = 8 \Rightarrow a_1 + 6d = 8 \Rightarrow -10d + 6d = 8$

$\Rightarrow d = -2 \cdots\cdots$(B)

$a_1 = -10d = -10 \times (-2) = 20 \cdots\cdots$(A)

(C) 設第 n 項開始小於 0

$\Rightarrow a_1 + (n-1)d < 0 \Rightarrow 20 + (n-1)(-2) < 0 \Rightarrow n > 11$

∴自第 12 項開始小於 0

(D) $a_1 + a_2 + \cdots + a_{21} = 0 \Rightarrow \frac{21}{2}(a_1 + a_{21}) = 0$

$\Rightarrow a_1 + a_{21} = 0$

13. (A) $m_{\overline{AB}} = \frac{-5-1}{2-6} = \frac{3}{2}$，$m_{\overline{BC}} = \frac{1-(-8)}{6-0} = \frac{3}{2}$

∵ $m_{\overline{AB}} = m_{\overline{BC}}$ ∴A、B、C 三點共線

(B) $m_{\overline{AB}} = \frac{3}{2}$，$m_{\overline{BD}} = \frac{1-(-3)}{6-12} = -\frac{2}{3}$

∵ $m_{\overline{AB}} \times m_{\overline{BD}} = \frac{3}{2} \times (-\frac{2}{3}) = -1$

∴ $\overline{AB} \perp \overline{BD} \Rightarrow \angle ABD = 90°$

(C) ∵ $m_{\overline{AB}} = \frac{3}{2}$ ∴根據點斜式，過 $P(0,3)$ 平行 $\overleftrightarrow{AB}$ 之直線方程式爲 $y - 3 = \frac{3}{2}(x-0) \Rightarrow 3x - 2y + 6 = 0$

(D) $\overline{AB}$ 中點爲 $(\frac{2+6}{2}, \frac{-5+1}{2}) = (4,-2)$

又 $m_{\overline{AB}} = \frac{3}{2} \Rightarrow$ 垂直平分線斜率爲 $-\frac{2}{3}$

∴ $\overline{AB}$ 的垂直平分線方程式爲

$y - (-2) = -\frac{2}{3}(x-4) \Rightarrow 2x + 3y - 2 = 0$

14. L 的斜率 $m_1 = \frac{4}{3}$，x 軸的斜率 $m_2 = 0$

$\Rightarrow \tan\theta = \pm\frac{\frac{4}{3} - 0}{1 + \frac{4}{3} \times 0} = \pm\frac{4}{3}$

∵ θ 是銳角∴ $\tan\theta = \frac{4}{3} \Rightarrow \cos\theta = \frac{3}{5}$

15. 設 L： $4x - 3y + k = 0$

圓： $(x-2)^2 + (y-1)^2 = 4 + 4 + 1 = 9$

∴圓心 $(2,1)$，半徑 $= 3$

L 和圓相切 ⇒ 圓心到 L 的距離 = 半徑

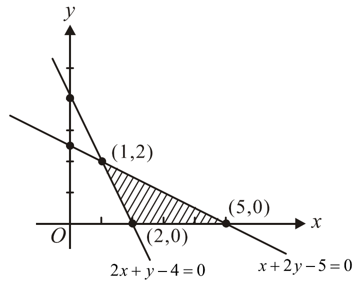
$\Rightarrow \frac{|4 \times 2 - 3 \times 1 + k|}{\sqrt{4^2 + (-3)^2}} = 3 \Rightarrow |k+5| = 15 \Rightarrow k + 5 = \pm 15$

$\Rightarrow k = -5 \pm 15 = 10$ 或 -20

∴L： $4x - 3y + 10 = 0$ 或 $4x - 3y - 20 = 0$

又 L 不過第二象限∴L： $4x - 3y - 20 = 0$

16. $\begin{cases} x \geq 0，y \geq 0 \\ 2x + y - 4 \geq 0 \\ x + 2y - 5 \leq 0 \end{cases}$ 的圖解爲



$\begin{cases} 2x + y - 4 = 0 \\ x + 2y - 5 = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 2 \end{cases}$

$f(x,y) = 3x + 2y$

$\Rightarrow f(2,0) = 3 \times 2 + 2 \times 0 = 6$

$f(5,0) = 3 \times 5 + 2 \times 0 = 15$ (最大值)

$f(1,2) = 3 \times 1 + 2 \times 2 = 7$

17. 拋物線 $(y-2)^2 = -4(x+1)$

$4c = -4 \Rightarrow c = -1 \Rightarrow$ 焦距 $= 1$

又頂點爲 $(-1,2)$，對稱軸爲 $y - 2 = 0$

⇒ 開口向左且焦點爲 $(-1-1,2) = (-2,2)$

∴以焦點 $(-2,2)$ 爲圓心，焦距 1 爲半徑的圓方程式爲

$(x+2)^2 + (y-2)^2 = 1$

18. 雙曲線 $4x^2 - 5y^2 - 16x + 20y - 24 = 0$

$\Rightarrow 4(x^2 - 4x + 2^2) - 5(y^2 - 4y + 2^2) = 24 + 16 - 20$

$\Rightarrow 4(x-2)^2 - 5(y-2)^2 = 20 \Rightarrow \frac{(x-2)^2}{5} - \frac{(y-2)^2}{4} = 1$

∴ $a^2 = 5$，$a = \sqrt{5}$

又 P 在雙曲線上，A、B 爲焦點

∴ $\left|\overline{PA} - \overline{PB}\right| = 2a = 2\sqrt{5}$

19. (A) $P^6_3 = \frac{6!}{3!} = 120$

(B) 每個球有 5 個箱子可選 $\Rightarrow 5 \times 5 \times 5 = 5^3 = 125$

(C) $H^4_6 = C^9_6 = \frac{9!}{6!3!} = 84$

(D) $\frac{6!}{6} = 5! = 120$

20. P(至少一人投進) $= 1 - P$(都不進)

$= 1 - (1 - \frac{2}{5})(1 - \frac{1}{4}) = 1 - \frac{3}{5} \times \frac{3}{4} = \frac{11}{20}$

21. 期望值 $= \frac{1}{6^2} \times 10 + 2 \times \frac{1 \times C^5_1}{6^2} \times 5 + \frac{5^2}{6^2} \times (-3)$

(兩個 6 點)　(恰一個 6 點)　(沒有 6 點)

$= \frac{10}{36} + \frac{50}{36} - \frac{75}{36} = -\frac{15}{36} = -\frac{5}{12}$

22. $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3-2x}}{x}$

$= \lim_{x \to 0} \frac{(\sqrt{3+x} - \sqrt{3-2x})(\sqrt{3+x} + \sqrt{3-2x})}{x(\sqrt{3+x} + \sqrt{3-2x})}$

$$= \lim_{x\to 0}\frac{(3+x)-(3-2x)}{x(\sqrt{3+x}+\sqrt{3-2x})} = \lim_{x\to 0}\frac{3x}{x(\sqrt{3+x}+\sqrt{3-2x})}$$

$$= \frac{3}{\sqrt{3}+\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

23. $f(x) = (x^3-2x^2-1)^5$

$\Rightarrow f'(x) = 5(x^3-2x^2-1)^4 \times (x^3-2x^2-1)'$

$= 5(x^3-2x^2-1)^4(3x^2-4x)$

$$\lim_{h\to 0}\frac{f(1+h)-f(1)}{5h} = \frac{1}{5}\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \frac{1}{5}f'(1)$$

$$= \frac{1}{5}\times[5(1-2-1)^4(3-4)] = -16$$

24. $\int_1^4 \frac{x^2+1}{\sqrt{x}}dx = \int_1^4 (\frac{x^2}{\sqrt{x}}+\frac{1}{\sqrt{x}})dx = \int_1^4 (x^{\frac{3}{2}}+x^{-\frac{1}{2}})dx$

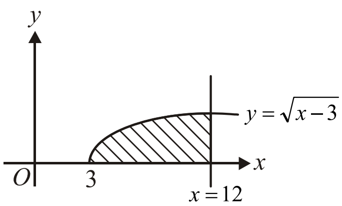
$$= \frac{2}{5}x^{\frac{5}{2}}+2x^{\frac{1}{2}}\Big|_1^4 = \frac{2}{5}\sqrt{x^5}+2\sqrt{x}\Big|_1^4$$

$$= \frac{2}{5}(\sqrt{4^5}-1)+2(\sqrt{4}-1) = \frac{62}{5}+2 = \frac{72}{5}$$

25. 如右圖所示

所求斜線區域面積

$= \int_3^{12}\sqrt{x-3}dx$



令 $u = x-3 \Rightarrow du = dx$

又 $x = 3 \Rightarrow u = 0$

$x = 12 \Rightarrow u = 9$

$$\therefore \int_3^{12}\sqrt{x-3}dx = \int_0^9\sqrt{u}du = \int_0^9 u^{\frac{1}{2}}du$$

$$= \frac{2}{3}u^{\frac{3}{2}}\Big|_0^9 = \frac{2}{3}\sqrt{u^3}\Big|_0^9 = \frac{2}{3}\sqrt{9^3} = 18$$