

## 105 學年度四技二專第一次聯合模擬考試 共同科目 數學(C)卷 詳解

數學(C)卷

105-1-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	A	A	C	D	D	A	C	C	B	A	B	D	B	B	B	C	A	D	D	C	D	A	B	A

1. 設  $P(x, y)$  由分點公式知：

$$x = \frac{2 \times 2 + 3 \times (-3)}{2+3} = -1, \quad y = \frac{2 \times (-5) + 3 \times (5)}{2+3} = 1$$

2.  $2y - 5x + 3 = 0 \Rightarrow 5x - 2y - 3 = 0$

直線斜率為  $\frac{5}{2}$ ，兩直線垂直

$\therefore$  所求之直線斜率為  $-\frac{2}{5}$

$$y - 1 = \frac{-2}{5}[x - (-3)] \Rightarrow 2x + 5y + 1 = 0$$

3. 設  $L: y = \frac{3}{4}x + k$ ， $x$  截距為  $\frac{-4}{3}k$ ， $y$  截距為  $k$

$$-\frac{4}{3}k + k = 2, \quad k = -6$$

4.  $\tan A = \frac{4}{3} = \frac{BC}{8}, \quad BC = \frac{32}{3}$

5. 同除以  $\cos \theta$

$$\text{原式} = \frac{\frac{2 \sin \theta + 3 \cos \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\cos \theta}} = \frac{2 \tan \theta + 3}{\tan \theta - 1} = \frac{2 \times \frac{3}{2} + 3}{\frac{3}{2} - 1} = 12$$

6.  $\sin \theta = -\frac{1}{2}, \Rightarrow \theta \in$  第三、四象限

且  $\cos \theta > 0, \Rightarrow \theta \in$  第一、四象限  
所以  $\theta$  在第四象限

$$\text{又 } \sin 330^\circ = -\frac{1}{2}, \text{ 所以 } \theta = 330^\circ$$

7.  $\cos 63^\circ = \sin 27^\circ, \cot 44^\circ = \tan 46^\circ > 1$

$\cos 350^\circ = \sin 80^\circ, \sin 27^\circ$  最小

8.  $\overrightarrow{BA} \cdot \overrightarrow{AC} = -\overrightarrow{AB} \cdot \overrightarrow{AC} = -|\overrightarrow{AB}| \times |\overrightarrow{AC}| \times \cos 60^\circ$

$$= -6 \times 10 \times \frac{1}{2} = -30$$

9.  $\overrightarrow{AB} = (-1, 2), \overrightarrow{AC} = (-3, -4)$

$$\cos A = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| \times |\overrightarrow{AC}|} = \frac{-5}{5\sqrt{5}} = \frac{-1}{\sqrt{5}}$$

10. 直線  $\overline{BC}$  的斜率  $m_{\overline{BC}} = 3$

直線  $\overline{BC}$  的為程式： $3x - y + 2 = 0$

$\overline{BC}$  邊上的高 =  $A$  點至  $\overline{BC}$  的距離

$$\frac{|3 \times 1 - 0 + 2|}{\sqrt{3^2 + (-1)^2}} = \frac{\sqrt{10}}{2}$$

11.  $\vec{a} + t\vec{b} = (-t+3, 2t+1), \vec{a} - \vec{b} = (4, -1)$

$$(-t+3, 2t+1) \cdot (4, -1) = 0, \quad t = \frac{11}{6}$$

12.  $\overrightarrow{AB} = (-3, 4), |\overrightarrow{AB}| = 5$

$$\vec{u} = -\frac{1}{5}\overrightarrow{AB} = \left(\frac{3}{5}, -\frac{4}{5}\right)$$

13.  $y = -2(x+1)^2 + 3 = -2x^2 - 4x + 1$

$$a = -4, \quad b = 1, \quad a + b = -3$$

14. 設點  $C(a, 0), \overline{AC} = \overline{BC}$

$$\Rightarrow \sqrt{(a+1)^2 + (0-9)^2} = \sqrt{(a-4)^2 + (0+6)^2}$$

$$a = -3$$

15. 直線  $M$  的斜率為  $-\frac{3}{2}$

因為平行斜率相等， $\frac{-2-1}{a-0} = -\frac{3}{2}, \quad a = 2$

16.  $\sin \alpha = \frac{4}{5}, \cos \alpha = \frac{3}{5}, \sin \beta = \frac{5}{13}, \cos \beta = \frac{-12}{13}$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \frac{3}{5} \times \frac{-12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{-16}{65}$$

17.  $f(x) = \sin^2 x - 2 \cos x + 1 = (1 - \cos^2 x) - 2 \cos x + 1$

$$= -\cos^2 x - 2 \cos x + 2 = -(\cos x + 1)^2 + 3$$

$\cos x = -1$  時，有最大值 3

18.  $(\sin \theta + \cos \theta)^2 = \left(\frac{3}{4}\right)^2$

$$1 + 2 \sin \theta \cos \theta = \frac{9}{16} \Rightarrow \sin \theta \cos \theta = \frac{-7}{32}$$

$$\tan \theta + \cot \theta = \frac{1}{\sin \theta \times \cos \theta} = -\frac{32}{7}$$

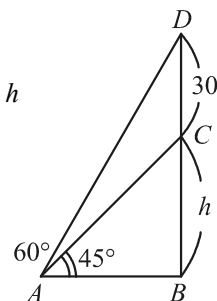
19. 如圖所示：

鐵塔高  $\overline{CD} = 30$ ，設山丘高  $\overline{BC} = h$

$$\tan 45^\circ = \frac{h}{AB} \Rightarrow \overline{AB} = h$$

$$\tan 60^\circ = \frac{h+30}{h} = \sqrt{3}$$

$$h = 15(\sqrt{3} + 1)$$



20. 由餘弦定理知：

$$\overline{BC}^2 = 2 + (4 + 2\sqrt{3}) - 2\sqrt{2}(\sqrt{3} + 1) \frac{1}{\sqrt{2}} = 4$$

$$\overline{BC} = 2$$

再由正弦定理：

$$\frac{2}{\sin 45^\circ} = \frac{\sqrt{2}}{\sin C} \Rightarrow \sin C = \frac{1}{2}$$

$\angle C = 30^\circ$  或  $150^\circ$  (不合)

$$\angle B = 180^\circ - 30^\circ - 45^\circ = 105^\circ$$

21.  $\vec{BC} = \vec{BA} + \vec{AC} = (0, 5)$

$$|\vec{AB}| = 5, |\vec{AC}| = \sqrt{10}, |\vec{BC}| = 5$$

$$\text{所求} = 10 + \sqrt{10}$$

22.  $2\vec{b} + 3\vec{c} = -\vec{a}$

$$\Rightarrow |2\vec{b} + 3\vec{c}| = |-\vec{a}|$$

$$\Rightarrow |2\vec{b} + 3\vec{c}|^2 = |-\vec{a}|^2$$

$$\Rightarrow |\vec{a}|^2 = 4|\vec{b}|^2 + 12|\vec{b}| \times |\vec{c}| \times \cos 60^\circ + 9|\vec{c}|^2$$

$$= 36 + 36 + 36 = 108$$

$$|\vec{a}| = \sqrt{108}$$

23. 設  $\vec{AD} = t$

$\triangle ABC$  面積 =  $\triangle ABD$  面積 +  $\triangle ACD$  面積

$$\frac{1}{2} \times 6 \times 10 \times \sin 60^\circ$$

$$= \frac{1}{2} \times 6t \times \sin 30^\circ + \frac{1}{2} \times 10t \times \sin 30^\circ$$

$$t = \frac{15\sqrt{3}}{4}$$

24.  $\tan 680^\circ = \tan(-40^\circ) = -\tan 40^\circ$

$$= -\tan 2 \times 20^\circ = \frac{2 \tan 20^\circ}{1 - \tan^2 20^\circ} = \frac{-2a}{1 - a^2}$$

25.  $y = ax^2 + bx + c$

開口向下： $a < 0$

頂點在第二象限

$$-\frac{b}{2a} < 0 \Rightarrow b < 0$$

由與  $y$  軸的交點知  $c < 0$

$$ax + by + c = 0$$

$x$	0	$-\frac{c}{a}$
$y$	$-\frac{c}{b}$	0

得到  $-\frac{c}{b} < 0$ ,  $-\frac{c}{a} < 0$  不通過第一象限

