

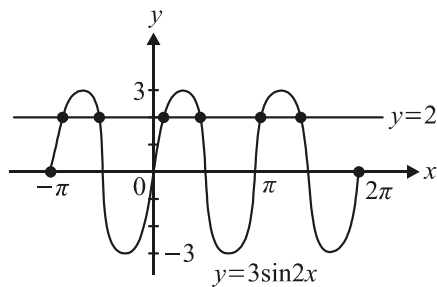
104 學年度四技二專第四次聯合模擬考試 共同科目 數學(C)卷 詳解

數學(C)卷

104-4-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	C	D	B	D	A	B	A	C	D	A	C	B	A	A	C	B	D	C	A	D	B	B	A	D

1. $\begin{vmatrix} 1 & 0 & 5 \\ 5 & 3 & 3 \\ 1 & 1 & 8 \end{vmatrix} = 24 + 25 + 0 - 15 - 3 - 0 = 31$
2. 設首項為 $a_1 = -10$ ，公差為 $d = 3$ ，且共有 n 項
則 $a_n = 44$ 代入 $a_n = a_1 + (n-1)d$
 $\Rightarrow 44 = (-10) + (n-1) \cdot 3$
 $\therefore n = 19$
共有 19 項的級數 $S_{19} = \frac{[(-10) + 44] \cdot 19}{2} = 323$
3. 設 m 為斜率
(A) 截距式 $\frac{x}{2} + \frac{y}{3} = 1 \Rightarrow m = -\frac{3}{2} < 0$
(B) 一般式 $4x + 5y + 6 = 0 \Rightarrow m = -\frac{4}{5} < 0$
(C) 點斜式 $(y+7) = -8(x+9) \Rightarrow m = -8 < 0$
(D) 斜截式 $y = 10x + 11 \Rightarrow m = 10 > 0$ ，斜率為正
4. $\sin 2010^\circ = \sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$
5. $y = 3\sin 2x \Rightarrow y$ 值之範圍為 $-3 \leq y \leq 3$ ，週期 π
如圖所示，在 $-\pi \leq x \leq 2\pi$ 範圍內共有 6 個交點

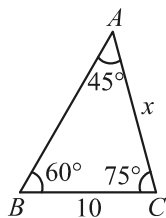


6. $\angle B = 180^\circ - 45^\circ - 75^\circ = 60^\circ$

$$\frac{x}{\sin 60^\circ} = \frac{10}{\sin 45^\circ}$$

$$x \cdot \frac{1}{\sqrt{2}} = 10 \cdot \frac{\sqrt{3}}{2}$$

$$x = 5\sqrt{6}$$



7. M 的坐標為 $(\frac{2+8}{2}, \frac{5+2}{2}) = (5, \frac{7}{2})$
 G 的坐標為 $(\frac{2+(-1)+8}{3}, \frac{5+(-1)+2}{3}) = (3, 2)$
 $\therefore A$ 的坐標為 $(2, 5)$
 $\therefore \vec{AM} = (3, -\frac{3}{2}), \vec{AG} = (1, -3)$

$$\vec{AM} \cdot \vec{AG} = 3 \cdot 1 + (-\frac{3}{2}) \cdot (-3) = 3 + \frac{9}{2} = \frac{15}{2}$$

8. $f(6) = 216 - 108 - 96 - 12 = 0$

$\therefore x - 6$ 為 $f(x)$ 因式

9. 重複使用綜合除法：

$$\begin{array}{r|l} 2 & -18 + 55 - 51 \\ & + 6 - 36 + 57 \\ \hline & \end{array} \Bigg| 3$$

$$\begin{array}{r|l} 2 & -12 + 19 \\ & + 6 - 18 \\ \hline & \end{array} \Bigg| +6$$

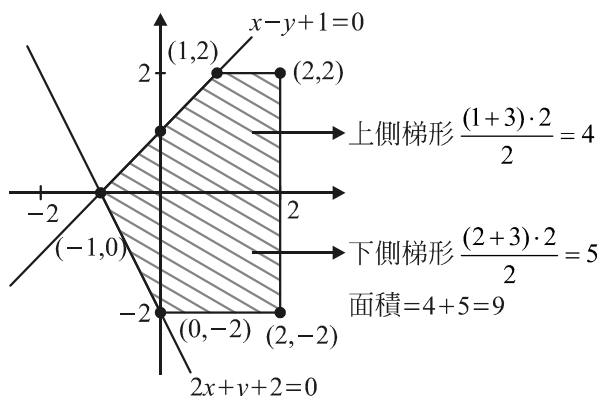
$$\begin{array}{r|l} 2 & -6 \\ & + 6 \\ \hline & \end{array} \Bigg| +1$$

$$2 \Big| + 0$$

$$a = 2, b = 0, c = 1, d = 6$$

$$a + 2b + 3c + 4d = 2 + 0 + 3 + 24 = 29$$

10.



11. 設兩根 $\alpha = 3 - 4i$ ， β 未知
根與係數關係可知 $\alpha + \beta = a$ ， $\alpha\beta = 2i + 11$
$$\beta = \frac{2i + 11}{\alpha} = \frac{2i + 11}{3 - 4i} = \frac{25 + 50i}{25} = 1 + 2i$$

$$a = \alpha + \beta = (3 - 4i) + (1 + 2i) = 4 - 2i$$
12. $10^2 \cdot (10^{2x-x^2}) = (10^{-2})^{\frac{3}{2}}$ ， $10^{2+2x-x^2} = 10^3$
 $2 + 2x - x^2 = 3 \Rightarrow x^2 - 2x + 1 = 0$
 $(x-1)^2 = 0$ ， $\therefore x = 1$
13. 底數 $\frac{1}{5} < 1$ ，若對數值越小，則其真數愈大
 $\therefore x - 1 < 3 - x \Rightarrow x < 2 \dots\dots \textcircled{1}$
同時，每個對數之真數都必須大於 0
 $\therefore \begin{cases} x - 1 > 0 \\ 3 - x > 0 \end{cases} \Rightarrow 1 < x < 3 \dots\dots \textcircled{2}$
取 $\textcircled{1}$ 、 $\textcircled{2}$ 兩式代表圖形之共同部分

得不等式解之範圍為 $1 < x < 2$

14. $f(x) = -x^2 - 3x + 3$

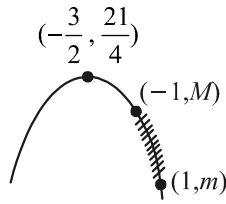
$$= -(x + \frac{3}{2})^2 + \frac{21}{4}$$

頂點 $(-\frac{3}{2}, \frac{21}{4})$ 不在

$-1 \leq x \leq 1$ 的範圍內

由圖形可知： $M = f(-1) = 5$

$$m = f(1) = -1, M + m = 4$$



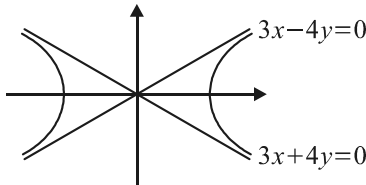
15. 漸近線 $3x \pm 4y = 0$ ，過原點的直線中，若其斜率為

m ，且使得 $-\frac{3}{4} < m < \frac{3}{4}$ 成立，則該直線和雙曲線有交

點，設 m_A 、 m_B 、 m_C 、 m_D 表選項(A)、(B)、(C)、(D)

中之直線斜率，則 $m_A = -\frac{2}{3}$ ， $m_B = -\frac{3}{4}$ ， $m_C = -\frac{4}{3}$ ，

$m_D = -\frac{5}{4}$ ，故選(A)



16. $C: x^2 + y^2 - 4x = 0 \Rightarrow (x-2)^2 + y^2 = 4$

圓心 $(2, 0)$ ，半徑 $r = 2$

$$L: (y-3) = m(x-4) \Rightarrow mx - y + 3 - 4m = 0$$

因為 C 和 L 相切，圓心和直線距離為 r

$$d(\text{心}, L) = \frac{|2m - 0 + 3 - 4m|}{\sqrt{m^2 + (-1)^2}} = 2$$

$$|-2m + 3| = 2\sqrt{m^2 + 1}, 4m^2 - 12m + 9 = 4m^2 + 4$$

$$m = \frac{5}{12}$$

17. $f'(x) = \frac{(-2) \cdot (3x+2) - (1-2x) \cdot 3}{(3x+2)^2} = -\frac{7}{(3x+2)^2}$

$$f'(-1) = -\frac{7}{(-1)^2} = -7$$

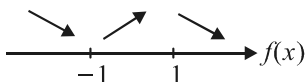
18. $\lim_{n \rightarrow \infty} \frac{1+2n+3n^2}{n^2-1} = 3, \lim_{n \rightarrow \infty} \frac{5n-4}{n+1} = 5$

兩數列的極限值存在，因此兩數列差的極限值，等於兩數列極限值的差：

$$\lim_{n \rightarrow \infty} \left[\frac{1+2n+3n^2}{n^2-1} - \frac{5n-4}{n+1} \right] = 3 - 5 = -2$$

19. $f(x) = -x^3 + 3x - 3$

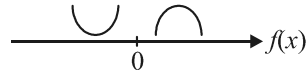
$$f'(x) = -3x^2 + 3 = -3(x+1)(x-1)$$



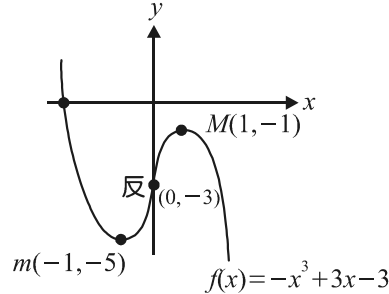
在 $x = -1$ 時，有相對極小值 $f(-1) = -5$

在 $x = 1$ 時，有相對極大值 $f(1) = -1$

$$f''(x) = -6x$$



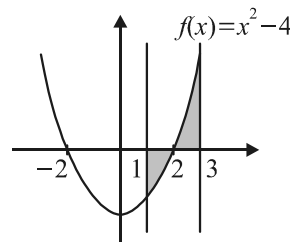
$f(0) = -3$ ，反曲點 $(0, -3)$ ，將極值點 $(-1, -5)$ 、 $(1, -1)$ 和反曲點 $(0, -3)$ 畫在坐標上觀察，可知 $f(x)$ 和 x 軸只有一個交點



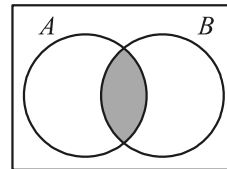
20. $\int_1^2 (x^2 - 4) dx = (\frac{x^3}{3} - 4x) \Big|_1^2 = -\frac{5}{3}$

$$\int_2^3 (x^2 - 4) dx = (\frac{x^3}{3} - 4x) \Big|_2^3 = \frac{7}{3}$$

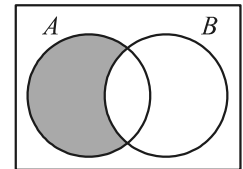
$$\text{面積} = \left| -\frac{5}{3} \right| + \frac{7}{3} = \frac{12}{3} = 4$$



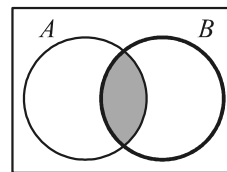
21. 利用凡氏圖觀察集合之間的機率關係



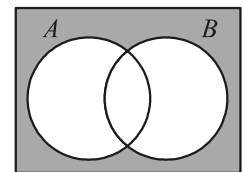
(A) $P(A \cap B)$



(B) $P(A - B)$



(C) $P(A | B)$



(D) $P(A' \cap B')$

(A) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{7}{12} = \frac{1}{4}$$

(B) $P(A - B) = P(A) - P(A \cap B) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

(C) $P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$

(D) $P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{7}{12} = \frac{5}{12}$

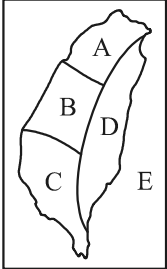
22. 三球同色可能為 3 白, 3 綠, 3 藍

$$P(3 \text{ 球同色}) = \frac{C_3^5 + C_3^4 + C_3^3}{C_3^{12}} = \frac{10+4+1}{220} = \frac{15}{220} = \frac{3}{44}$$

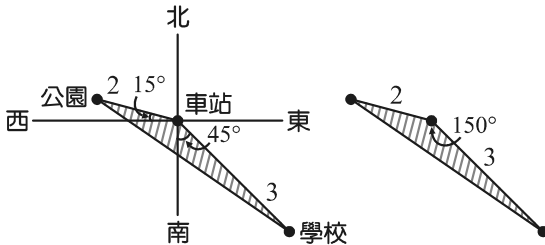
23. 相鄰區域多的先著色

$E \rightarrow B \rightarrow D \rightarrow A \rightarrow C$

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 2 = 240$$



24. 三角形住宅區的面積為 $\Delta = \frac{1}{2} \cdot 2 \cdot 3 \cdot \sin 150^\circ = \frac{3}{2}$



25. 根據常態分配 68-95-99.7 原則

35 分以下約占全體之 16%

共計 $1000 \times 16\% = 160$ (人)

$$\therefore 3 < \frac{160}{50} = 3.2 < 4$$

共需 4 班才能容納

