104 學年度四技二專第二次聯合模擬考試
共同科目　數學(C)卷　詳解

數學(C)卷 104-2-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	C	C	B	C	D	C	C	A	C	A	B	B	A	D	B	B	D	B	C	D	A	B	D	A

1. $\begin{vmatrix} 2015 & 2014 \\ 1221 & 1222 \end{vmatrix} = \begin{vmatrix} 1 & 2014 \\ -1 & 1222 \end{vmatrix} = 1222 + 2014 = 3236$

2. $3x = x - 2 \Rightarrow x = -1$，$7i = yi + 5i \Rightarrow y = 2$
$\therefore x + y = 1$

3. $x = 1$ 代入 $(3x^3 + 5x^2 - 2x - 7)^3 (x+1)^2$ 得 -4

4. ΔABC 之面積 $= \dfrac{1}{2} \left\| \begin{matrix} 5 & 1 \\ -1 & -3 \end{matrix} \right\| = \dfrac{1}{2} |-15 - (-1)| = 7$

5. $2x - 3 > 4$ 或 $2x - 3 < -4 \Rightarrow x > \dfrac{7}{2}$ 或 $x < -\dfrac{1}{2}$
所以 $a = 4$，$b = -1$，$a + b = 3$

6. 設平行 $x - 2y = 0$ 之直線方程式為 $x - 2y + k = 0$，代入交點 $(-\dfrac{2}{3}, 2)$，得 $k = \dfrac{14}{3}$，所以 $3x - 6y + 14 = 0$

7. 開口向下所以 $a < 0$；與 y 軸交點的 y 座標 $(0, c)$，所以 $c < 0$；與 x 軸交兩點，所以判別式 $b^2 - 4ac$ 大於 0；頂點 $(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a})$ 在第二象限，且 $a < 0$ 所以 $b < 0$

8. $x = \sqrt{1^2 + \sqrt{3}^2} - 5 = 2 - 5 = -3$
$y = -\sqrt{1^2 + \sqrt{3}^2} - 5 = -7$
所以 $x - 2y = -3 + 14 = 11$

9. $\begin{vmatrix} x+a & y+b \\ 5c & 5d \end{vmatrix} = 25 \Rightarrow \begin{vmatrix} x+a & y+b \\ c & d \end{vmatrix} = 5$
又 $\begin{vmatrix} 2x & 2y \\ 3c & 3d \end{vmatrix} = 12 \Rightarrow \begin{vmatrix} x & y \\ c & d \end{vmatrix} = 2$
$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} x+a & y+b \\ c & d \end{vmatrix} - \begin{vmatrix} x & y \\ c & d \end{vmatrix} = 5 - 2 = 3$

10. $|z| = \left| \dfrac{(12-5i)(5+7i)(-2+3i)}{(3+2i)(-7+5i)(3-4i)} \right|$
$= \dfrac{|12-5i||5+7i||-2+3i|}{|3+2i||-7+5i||3-4i|} = \dfrac{13}{5}$

11. 因為 θ 為第四象限角，所以 $\sin\theta = -\dfrac{3}{5}$
$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times (-\dfrac{3}{5}) \times \dfrac{4}{5} = -\dfrac{24}{25}$

12. 令多項式 $f(x) = (x^2 + x - 20)Q(x) + (3x + 2)$
$= (x+5)(x-4)Q(x) + (3x+2)$
多項式 $f(x)$ 除以 $x + 5$ 所得的餘式為
$f(-5) = 3(-5) + 2 = -15 + 2 = -13$

13. $\overline{AB} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$，$\overline{AC} = \sqrt{1^2 + 3^2} = \sqrt{10}$
$\overline{BC} = \sqrt{3^2 + 1^2} = \sqrt{10}$，所以 $\overline{AC} = \overline{BC}$
故菱形為 $ACBD$，則 $D(-1, 3)$ 在第二象限

14. 利用餘弦定理 $b^2 = 2^2 + (\sqrt{3}+1)^2 - 2 \cdot 2 \cdot (\sqrt{3}+1)\cos 30^\circ$
$= 4 + 3 + 2\sqrt{3} + 1 - 4(\sqrt{3}+1) \cdot \dfrac{\sqrt{3}}{2} = 2$
所以 $b = \sqrt{2}$
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{2 + 3 + 2\sqrt{3} + 1 - 4}{2 \times \sqrt{2} \times (\sqrt{3}+1)} = \dfrac{1}{\sqrt{2}}$
所以 $\angle A = 45^\circ$，$\angle C = (180 - 30 - 45)^\circ = 105^\circ$
ΔABC 為鈍角三角形

15. 連續利用綜合除法將左式除以 $x + 1$

$$\begin{array}{l|l} 1+3-2+3+6 & -1 \\ \quad -1-2+4-7 & \\ \hline 1+2-4+7 & -1 \\ \quad -1-1+5 & \\ \hline 1+1-5 & +12 \\ \quad -1+0 & \\ \hline 1+0 & -5 \\ \quad -1 & \\ \hline 1-1 & \end{array}$$

依序可求出 $a = 1$，$b = -1$，$c = -5$，$d = 12$，$e = -1$

16. $\begin{vmatrix} x-1 & x & x+1 \\ x+1 & x-1 & x \\ x & x+1 & x-1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 3x & 3x & 3x \\ x+1 & x-1 & x \\ x & x+1 & x-1 \end{vmatrix} = 0$
$\Rightarrow 3x \begin{vmatrix} 1 & 1 & 1 \\ x+1 & x-1 & x \\ x & x+1 & x-1 \end{vmatrix} \times (-x) = 0$
$\Rightarrow 3x \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0 \Rightarrow 9x = 0 \Rightarrow x = 0$

17. $\dfrac{(\cos 24^\circ + i\sin 24^\circ)^2 (\cos 307^\circ + i\sin 307^\circ)}{(\cos 29^\circ + i\sin 29^\circ)^5}$
$= \cos(24^\circ \times 2 + 307^\circ - 29^\circ \times 5) + i\sin(24^\circ \times 2 + 307^\circ - 29^\circ \times 5)$
$= \cos 210^\circ + i\sin 210^\circ = -\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$

18. $\omega = \dfrac{-1 + \sqrt{3}i}{2} \Rightarrow 2\omega + 1 = \sqrt{3}i \Rightarrow \omega^2 + \omega + 1 = 0$

$\therefore \omega^3 = 1$

$\omega^{67} + \omega^{68} + \omega^{69} + \cdots + \omega^{2015} = \omega + \omega^2 + 1 + \cdots + \omega^2$

$= \omega + \omega^2 = -1$

19. $ax^2 + 4x + b > 0$ 的解爲 $-\frac{1}{2} < x < \frac{3}{2}$

$\Rightarrow (x+\frac{1}{2})(x-\frac{3}{2}) < 0 \Rightarrow (2x+1)(2x-3) < 0$

$\Rightarrow 4x^2 - 4x - 3 < 0 \Rightarrow -4x^2 + 4x + 3 > 0$

比較 $ax^2 + 4x + b > 0 \Rightarrow a = -4$，$b = 3$

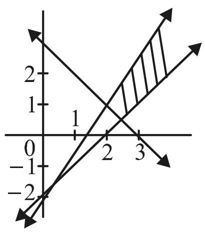
$\therefore a + b = -4 + 3 = -1$

20. $(\sqrt{x}^2 + \sqrt{y}^2 + \sqrt{z}^2)(\sqrt{\frac{1}{x}}^2 + \sqrt{\frac{4}{y}}^2 + \sqrt{\frac{25}{z}}^2)$

$\geq (\sqrt{x}\sqrt{\frac{1}{x}} + \sqrt{y}\sqrt{\frac{4}{y}} + \sqrt{z}\sqrt{\frac{25}{z}})^2$

$\Rightarrow (4)(\frac{1}{x} + \frac{4}{y} + \frac{25}{z}) \geq (1+2+5)^2 \Rightarrow \frac{1}{x} + \frac{4}{y} + \frac{25}{z} \geq 16$

21. $\begin{cases} x + y = 3 \\ 3x - 2y = 4 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 1 \end{cases}$



$\begin{cases} x + y = 3 \\ x - y = 2 \end{cases} \Rightarrow \begin{cases} x = \frac{5}{2} \\ y = \frac{1}{2} \end{cases}$

$x + 2y$ 最小値 $= \frac{5}{2} + 2 \times \frac{1}{2} = \frac{7}{2}$

22. $x^2 - 3x + 1 = 0 \Rightarrow x - 3 + \frac{1}{x} = 0 \Rightarrow x + \frac{1}{x} = 3$

$x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2 = 9 - 2 = 7$

23. $\tan\alpha + \tan\beta = 5$，$\tan\alpha \cdot \tan\beta = 2$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \frac{5}{1-2} = -5$

$\sin^2(\alpha+\beta) = (\pm\frac{5}{\sqrt{1^2+5^2}})^2 = \frac{25}{26}$

$\begin{vmatrix} 1 & 0 & \sin(\alpha+\beta) \\ 0 & 1 & 0 \\ \sin(\alpha+\beta) & 2 & 1 \end{vmatrix} = 1 - \sin^2(\alpha+\beta) = \frac{1}{26}$

$\therefore a = 1$，$b = 26$，$a + b = 27$，爲 3 的倍數

24. $\because \overrightarrow{a} - \overrightarrow{b} = (\sin\theta - \cos\theta, -1)$

$|\overrightarrow{a} - \overrightarrow{b}| = \sqrt{(\sin\theta - \cos\theta)^2 + (-1)^2}$

$= \sqrt{2 - 2\sin\theta\cos\theta}$ ……(1)

又 $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

$\Rightarrow (\frac{\sqrt{3}}{2})^2 = 1 + 2\sin\theta\cos\theta$

$\Rightarrow 2\sin\theta\cos\theta = -\frac{1}{4}$ 代入(1)

得 $|\overrightarrow{a} - \overrightarrow{b}| = \sqrt{2 - (-\frac{1}{4})} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

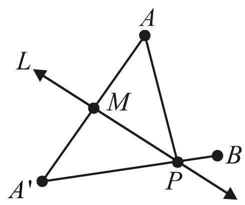
25. (1) 令 $A(2, 3)$ 相對於直線 L：$2x + 3y = 0$ 的對稱點爲 $A'(x', y')$，令 $\overleftrightarrow{AA'}$：$3x - 2y + k = 0$

將 $A(2, 3)$ 帶入得 $k = 0 \Rightarrow 3x - 2y = 0$

(2) 求 L 與 $\overleftrightarrow{AA'}$ 之交點 $M(x, y)$

$\Rightarrow \begin{cases} 2x + 3y = 0 \\ 3x - 2y = 0 \end{cases}$

$\Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow M(0, 0)$



(3) 又 $M(0, 0)$ 爲 $\overline{AA'}$ 之中點

$\Rightarrow \begin{cases} 0 = \frac{2 + x'}{2} \\ 0 = \frac{3 + y'}{2} \end{cases} \Rightarrow \begin{cases} x' = -2 \\ y' = -3 \end{cases} \Rightarrow A'(-2, -3)$

(4) $\overline{PA} + \overline{PB} = \overline{PA'} + \overline{PB}$ 的最小值應爲 $A' - P - B$ 三點共線，最小值 $= \overline{A'B} = \sqrt{(-2-5)^2 + (-3+2)^2}$

$= \sqrt{50} = 5\sqrt{2}$