

102 學年四技二專第四次聯合模擬考試 共同科目 數學(C)卷 詳解

數學(C)卷

102-4-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	D	D	C	B	A	D	C	B	C	C	A	A	C	D	D	D	B	A	B	A	C	B	C	B

1. $\begin{cases} ab > 0 \\ a+b < 0 \end{cases} \Rightarrow \begin{cases} a < 0 \\ b < 0 \end{cases}, \therefore \sqrt[3]{a} < 0 \text{ 且 } ab^4 < 0$

\therefore 點 $A(\sqrt[3]{a}, ab^4)$ 在第三象限

2. 令 $L': 3x+2y+k=0$

$$d(A, L') = \frac{|3 \times 3 + 2 \times 0 + k|}{\sqrt{3^2 + 2^2}} = \sqrt{13}$$

$|9+k|=13, \therefore k=4 \text{ 或 } -22$

L' 的 y 截距為 -2 或 11

3. $\tan \alpha = 1, \tan \beta = \frac{1}{2}$

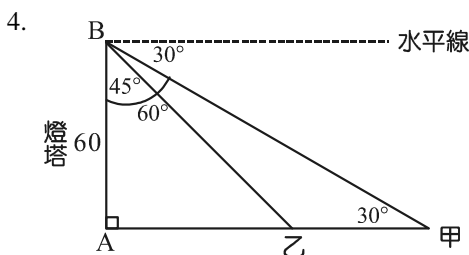
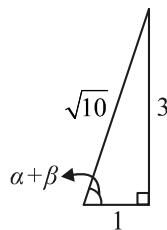
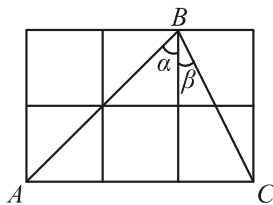
$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{1 + \frac{1}{2}}{1 - 1 \times \frac{1}{2}} = 3$$

由圖可知 $45^\circ < \alpha + \beta < 90^\circ$

$\therefore \sin(\angle ABC) = \sin(\alpha + \beta)$

$$= \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$



$\overline{A\text{甲}} = 60\sqrt{3}, \overline{AZ} = 60, \therefore \text{甲}\overline{Z} = 60\sqrt{3} - 60 \div 43.92$

5. $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = (8, 8) - (-4, 3) = (12, 5)$

$$|\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{AC}| = |(-4, 3)| + |(12, 5)| + |(8, 8)|$$

$$= 5 + 13 + 8\sqrt{2} = 18 + 8\sqrt{2} = x + 8\sqrt{2}$$

$\therefore x=18$ 為 6 的倍數

6. 令 $f(x) = (x+3)^{20} + 16 = (x+2)^2 Q(x) + ax + b$

$x = -2$ 代入得 $(-2+3)^{20} + 16 = 0 + a \times (-2) + b$

$17 = -2a + b \dots\dots ①$

$(x+3)^{20} + 16 = (x+2)^2 Q(x) + ax + b$ 兩邊微分，得

$$20(x+3)^{19} = 2(x+2)Q(x) + (x+2)^2 Q'(x) + a$$

$x = -2$ 代入，得 $a = 20$

$a = 20$ 代入 ①，得 $b = 57$

$\therefore a - b = 20 - 57 = -37$

7. $x = \sin 140^\circ + i \cos 140^\circ = \sin 40^\circ - i \cos 40^\circ$
 $= \cos 50^\circ - i \sin 50^\circ = \cos(-50^\circ) + i \sin(-50^\circ)$
 $= \cos(-50^\circ + 360^\circ) + i \sin(-50^\circ + 360^\circ)$
 $= \cos 310^\circ + i \sin 310^\circ$

$\therefore \text{Arg}(x) = 310^\circ$

8. 由柯西不等式可知

$$[a^2 + b^2 + (3c)^2][1^2 + (-3)^2 + (\frac{1}{3})^2] \geq (a - 3b + c)^2$$

則 $9 \times \frac{91}{9} \geq (a - 3b + c)^2, \therefore -\sqrt{91} \leq a - 3b + c \leq \sqrt{91}$

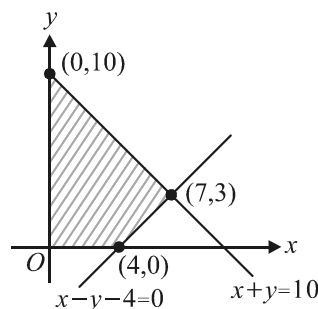
$a - 3b + c$ 最大值為 $\sqrt{91}$

9. $a < 0$ 且 $ax^2 + ax - 1 < 0$ ，判別式： $a^2 - 4 \times a \times (-1) < 0$

$a^2 + 4a < 0, a(a+4) < 0, -4 < a < 0$

$\therefore a = -1, -2, -3$ ，有 3 個解

10.



(x, y)	$(0, 0)$	$(0, 10)$	$(7, 3)$	$(4, 0)$
$x - 2y$	0	-20	1	4
				↑
				Max

11. $8 = (x+1) \begin{vmatrix} 1 & 2 & 5 \\ 1 & 3 & 6 \\ 1 & 4 & 9 \end{vmatrix} = (x+1) \begin{vmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & 2 & 4 \end{vmatrix}$

$$= (x+1) \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 2(x+1), \therefore x = 3$$

12. 令首項為 a ，公差為 d

$-3 = a_1 + a_2 + a_3 = [2a + (3-1) \times d] \times \frac{3}{2} \dots\dots ①$

$-20 = a_1 + a_2 + \dots + a_5 = [2a + (5-1) \times d] \times \frac{5}{2} \dots\dots ②$

由 ①、② 知 $a = 2, d = -3$

$\therefore \sum_{i=1}^6 a_i = [2a + (6-1) \times d] \times \frac{6}{2} = [4 + 5 \times (-3)] \times 3 = -33$

13. 令首項 $a = 2013$ ，公比 $r = 2013$ ，項數 $n = 102$

$$S = \frac{a(r^n - 1)}{r - 1} = \frac{2013[2013^{102} - 1]}{2012}, \therefore x = 102$$

14. $\sqrt[3]{1} < \sqrt[3]{3} < \sqrt[3]{8}$, $\therefore 1 < \sqrt[3]{3} < 2$, $\therefore 1 < a < 2$

$$b = (0.1)^{\sin 30^\circ} = 0.1^{\frac{1}{2}} = \sqrt{0.1} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}} < 1$$

$$c = \left(\frac{1}{5}\right)^{-5} = (5^{-1})^{-5} = 5^5 > 2, \therefore c > a > b$$

15. 真數 > 0 : $\begin{cases} 2x+1 > 0 \\ 5-x > 0 \end{cases} \Rightarrow \begin{cases} x > -\frac{1}{2} \\ x < 5 \end{cases}$

$$\log_3(2x+1) > \log_3(5-x) \Rightarrow 2x+1 > 5-x$$

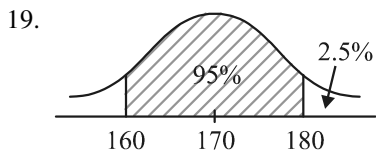
$$\therefore 3x > 4 \Rightarrow x > \frac{4}{3}, \text{ 故 } \frac{4}{3} < x < 5, a = \frac{4}{3}, b = 5$$

16. 全部 - 違規 (A、B 相鄰) = $5! - 4! \times 2 = 72$ 種

↑
A、B 可互換

17. $C_1^5 \times C_2^4 \times C_2^2 = 5 \times 6 \times 1 = 30$ 種

18. $P = \frac{\frac{1}{2} \times \frac{4}{5}}{\frac{1}{2} \times \frac{4}{5} + \frac{1}{2} \times \frac{3}{5}} = \frac{4}{7}$



$$600 \times 2.5\% = 15$$

20. C_1 圓心 $(0, 0)$, C_2 圓心 $(-4, 3)$

可知兩圓心距離為 5, 且 C_1 半徑 = 3

$$\therefore C_2 \text{ 半徑為 } 5 - 3 = 2 (\because \text{外切}), \therefore k = 2^2 = 4$$

21. $y = 3\left(x^2 + \frac{2}{3}x\right) + 29 = 3\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9}\right] + 29$

$$= 3\left(x + \frac{1}{3}\right)^2 + \frac{86}{3}$$

$$\therefore \left(x + \frac{1}{3}\right)^2 = \frac{1}{3}\left(y - \frac{86}{3}\right), 4c = \frac{1}{3}$$

$$\therefore \text{正焦弦長} = |4c| = \frac{1}{3}$$

22. 原式 = $\lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 + 3n} - n)(\sqrt{n^2 + 3n} + n)}{\sqrt{n^2 + 3n} + n}$

$$= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n) - n^2}{\sqrt{n^2 + 3n} + n} = \lim_{n \rightarrow \infty} \frac{3}{\sqrt{1 + \frac{3}{n}} + 1} = \frac{3}{1 + 1} = \frac{3}{2}$$

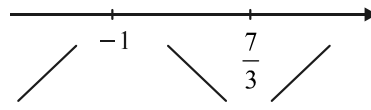
23.

球號	1	2	
正反面	1 正	1 正 1 反	2 正
(機率) P	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{4}$

$$E = \frac{1}{4} \times 1 + \frac{1}{4} \times 1 + \frac{1}{8} \times 2 = \frac{3}{4}$$

24. $f'(x) = 3x^2 - 4x - 7 = (3x - 7)(x + 1) = 0$

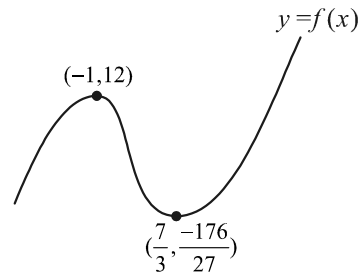
$$\therefore x = \frac{7}{3}, -1$$



$$f(-1) = -1 - 2 + 7 + 8 = 12$$

$f(x)$ 在 $x = -1$ 時, 有極大值 12

$$\therefore a = -1, b = 12, a + b = -1 + 12 = 11$$



25. 令 $x^2 + 2 = u$, $2x dx = du$

x	0	1
u	2	3

$$\text{原式} = \int_2^3 u^4 du = \frac{1}{5} u^5 \Big|_2^3 = \frac{1}{5} (3^5 - 2^5) = \frac{211}{5}$$