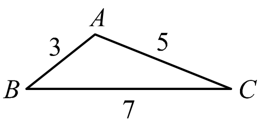
109 學年度四技二專第三次聯合模擬考試
共同科目　數學(B)卷　詳解

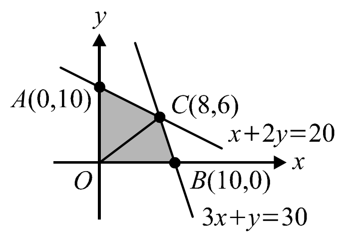
數學(B)卷 109-3-B

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	D	B	A	A	C	B	C	D	B	C	C	C	B	D	C	A	B	D	C	A	D	A	B	D

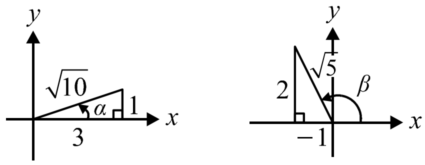
1. $m_{L1}=\frac{1}{5}$，$m_{L2}=-\frac{1}{2}$，$m_{L3}=1$，$m_{L4}=-2$
   $\because \overline{AB}$、$\overline{CD}$ 的斜率為正且 $\overline{AB}$ 比較平緩
   $\therefore m_{\overline{AB}}=\frac{1}{5}$
   $\Rightarrow$ 直線 AB 之方程式可能為 $x-5y+10=0$，故選(A)
2. 將 $y=2^x$ 之圖形上移 2 單位即得 $y=2^x+2$，故選(D)
3. $\overrightarrow{OC}=(1,3)+t(-2,1)=(1-2t,3+t)$
   $\Rightarrow |\overrightarrow{OC}|=\sqrt{(1-2t)^2+(3+t)^2}$
   $=\sqrt{1-4t+4t^2+9+6t+t^2}$
   $=\sqrt{5t^2+2t+10}=\sqrt{5(t+\frac{1}{5})^2+\frac{49}{5}}$
   $\therefore$ 當 $t=-\frac{1}{5}$ 時，$|\overrightarrow{OC}|$ 的最小值為 $\sqrt{\frac{49}{5}}=\frac{7\sqrt{5}}{5}$
   故選(B)
4. $\sin 1200°=\sin 120°=\sin 60°=\frac{\sqrt{3}}{2}$
   $\cos(-\frac{11\pi}{6})=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$
   $\sin 270°=-1$，$\cos 4\pi=1$
   $\therefore$ 原式 $=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}+(-1)\times 1=-\frac{1}{4}$，故選(A)
5. 由正弦定理知：$\frac{7}{\sin A}=\frac{5}{\sin B}=\frac{3}{\sin C}$



   令 $\sin A=7k$，$\sin B=5k$，$\sin C=3k$，$k\neq 0$
   $\therefore \frac{\sin C}{\sin A+\sin B}=\frac{3k}{7k+5k}=\frac{1}{4}$，故選(A)
6. $S_{n+2}=\frac{(n+2)(5+35)}{2}=240$
   $\Rightarrow 20(n+2)=240 \Rightarrow n+2=12 \Rightarrow n=10$，故選(C)
7. $\log_3 a=5 \Rightarrow a=3^5$
   $\log_3 b=7 \Rightarrow b=3^7$
   $\therefore \log(a+b)=\log(3^5+3^7)$
   $=\log[3^5\times(1+3^2)]=\log[3^5\times 10]=\log 3^5+\log 10$
   $=5\times\log 3+1=5\times 0.4771+1=3.3855$，故選(B)
8. 機率 $=\dfrac{C_1^4\times C_3^3\times 2!}{\frac{C_3^6\times C_3^3}{2!}\times 2!}=\frac{8}{20}=\frac{2}{5}$，故選(C)
9. 令 $x=-7$
   $\therefore f(x)=x^5+20x^4+99x^3+86x^2+288x+550$
   所求 $=f(x)$ 除以 $(x+7)$ 之餘式
   $$\begin{array}{rrrrrr|l} 1 & +20 & +99 & +86 & +288 & +550 & -7 \\ & -7 & -91 & -56 & -210 & -546 & \\ \hline 1 & +13 & +8 & +30 & +78 & +4 & \end{array}$$
   $\therefore f(-7)=4$，故選(D)
10. $\begin{vmatrix} 5a & b+2a & c+3b \\ 5d & e+2d & f+3e \\ 5g & h+2g & k+3h \end{vmatrix}=5\times\begin{vmatrix} a & b+2a & c+3b \\ d & e+2d & f+3e \\ g & h+2g & k+3h \end{vmatrix}$
    $\overset{(第1行\times(-2)加至第2行)}{=\!=\!=}5\times\begin{vmatrix} a & b & c+3b \\ d & e & f+3e \\ g & h & k+3h \end{vmatrix}$
    $\overset{(第2行\times(-3)加至第3行)}{=\!=\!=}5\times\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix}=5\times 5=25$
    故選(B)
11. 二元一次聯立不等式的圖形如下圖，連接 $\overline{CO}$



    區域面積 $=\Delta OAC+\Delta OBC$
    $=\frac{10\times 8}{2}+\frac{10\times 6}{2}=40+30=70$，故選(C)
12. 順序保持不變，其取法可視為 $AABBBCC$ 的排列方法數　$\therefore$ 取法有 $\frac{7!}{2!3!2!}=210$ (種)，故選(C)
13. $m_{L_1}=\frac{-7}{4}$、$m_{L_2}=\frac{-3}{a}$、$m_{L_3}=\frac{-2}{3}$
    (1) 若 $L_1\perp L_2 \Rightarrow \frac{-7}{4}\times\frac{-3}{a}=-1 \Rightarrow a=-\frac{21}{4}$
    (2) 若 $L_2\perp L_3 \Rightarrow \frac{-3}{a}\times\frac{-2}{3}=-1 \Rightarrow a=-2$
    (3) $\because \frac{-7}{4}\times\frac{-2}{3}\neq -1$　$\therefore L_1 \not\perp L_3$
    由上可知：$-\frac{21}{4}+(-2)=-\frac{29}{4}$，故選(C)

14.


$\because 90° < \alpha + \beta < 270°$

$\therefore$取 $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{1}{\sqrt{10}} \times \frac{-1}{\sqrt{5}} + \frac{3}{\sqrt{10}} \times \frac{2}{\sqrt{5}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \alpha + \beta = \frac{3\pi}{4}$，故選(B)

15. $0.2\overline{a5} = 0.2a5a5a5\cdots\cdots = 0.2 + 0.0a5 + 0.000a5\cdots\cdots$

$= 0.2 + \frac{0.0a5}{1-0.01} = \frac{2}{10} + \frac{a5}{990} = \frac{2a5-2}{990} = \frac{2a3}{990}$

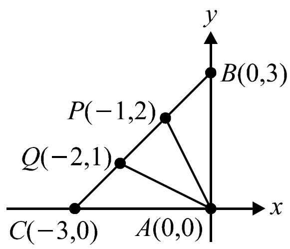
$\because \frac{25}{99} \le \frac{2a3}{990} < \frac{29}{99}$　$\therefore \frac{250}{990} \le \frac{2a3}{990} < \frac{290}{990}$

$\Rightarrow a = 5$ 、6、7、8，共 4 個，故選(D)

[另解] $\frac{25}{99} \le \frac{2a5-2}{990} < \frac{29}{99} \Rightarrow \frac{250}{990} \le \frac{2a3}{990} < \frac{290}{990}$

$\therefore a = 5$ 、6、7、8，共 4 個，故選(D)

16. (1) 若 C 與 A 、 E 同色：$5\times4\times1\times4\times1 = 80$ (種)
(2) 若 C 不與 A 、 E 同色：$5\times4\times3\times3\times1 = 180$ (種)
$\therefore 80 + 180 = 260$ (種)，故選(C)

17.


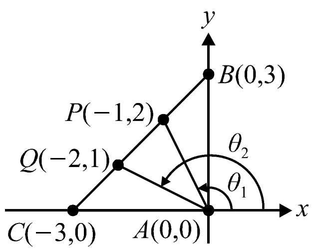
$\overrightarrow{AP} = (-1, 2) \Rightarrow |\overrightarrow{AP}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$\overrightarrow{AQ} = (-2, 1) \Rightarrow |\overrightarrow{AQ}| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

$\cos(\angle PAQ) = \frac{\overrightarrow{AP}\cdot\overrightarrow{AQ}}{|\overrightarrow{AP}||\overrightarrow{AQ}|} = \frac{(-1,2)\cdot(-2,1)}{\sqrt{5}\times\sqrt{5}} = \frac{4}{5}$

$\therefore \tan(\angle PAQ) = \frac{3}{4}$，故選(A)

[另解]



$\tan\angle PAQ = \tan(\theta_2 - \theta_1)$

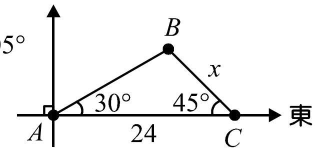
$= \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2\tan\theta_1} = \frac{-\frac{1}{2} - (-2)}{1 + (-\frac{1}{2})(-2)} = \frac{3}{4}$，故選(A)

18. 由餘式定理知：$f(-1) = 3$ 且 $f(2) = 6$
令 $(x+2)f(x) = (x^2 - x - 2)\cdot Q(x) + ax + b$
$\because f(-1) = 3$
$\therefore (-1+2)f(-1) = -a + b \Rightarrow -a + b = 3$ ……①
又 $f(2) = 6$
$\therefore (2+2)f(2) = 2a + b \Rightarrow 2a + b = 24$ ……②
解①、②得：$a = 7$ ，$b = 10$
$\therefore$餘式爲 $7x + 10$ ，故選(B)

19. 機率 $P = \frac{1}{3}\times\frac{2}{8} + \frac{1}{3}\times\frac{3}{6} + \frac{1}{3}\times\frac{1}{5} = \frac{19}{60}$ ，故選(D)

20. $\because 95 - 52 = 43$ 且 $10 - 52 = -42$　$\therefore 43 + (-42) = 1$
$\Rightarrow$ 刪去這 2 個成績會使平均下降
另外，刪去兩端之極值，會使資料更集中
$\therefore$標準差也會下降
由上可知，(C)爲正解

21. 如右圖所示，令 $\overline{BC} = x$
$\angle B = 180° - 30° - 45° = 105°$
由正弦定理知：

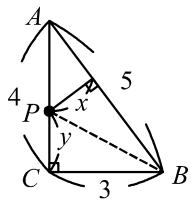


$\frac{24}{\sin 105°} = \frac{x}{\sin 30°}$

$\Rightarrow x \times \sin 105° = 24 \times \sin 30° \Rightarrow x \times \sin 75° = 24 \times \sin 30°$

$\Rightarrow x \times \frac{\sqrt{6}+\sqrt{2}}{4} = 24 \times \frac{1}{2} \Rightarrow x = 12(\sqrt{6} - \sqrt{2})$ ，故選(A)

22. 連接 $\overline{PB}$
ΔABC 面積
$= \Delta APB$ 面積 $+ \Delta BCP$ 面積



$\Rightarrow \frac{3\times4}{2} = \frac{5x}{2} + \frac{3y}{2} \Rightarrow 5x + 3y = 12$

由柯西不等式知

$(x^2 + y^2)(5^2 + 3^2) \ge (5x + 3y)^2 \Rightarrow 34(x^2 + y^2) \ge 12^2$

$\Rightarrow x^2 + y^2 \ge \frac{144}{34} = \frac{72}{17}$　$\therefore x^2 + y^2$ 之最小值爲 $\frac{72}{17}$

故選(D)

23. (1) $\because \alpha$ 、 β 爲 $x^2 - 6x + 1 = 0$ 之兩根
$\therefore \alpha^2 - 6\alpha + 1 = 0$ 且 $\beta^2 - 6\beta + 1 = 0$
$\Rightarrow \alpha^2 - 6\alpha = -1$ 且 $\beta^2 - 6\beta = -1$
(2) $\because \alpha$ 、 β 爲 $x^2 - 6x + 1 = 0$ 之兩根

$\therefore \begin{cases} \alpha + \beta = 6 \\ \alpha\beta = 1 \end{cases}$ (根與係數關係)

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 6^2 - 2 = 34$

由(1)(2)知 $\frac{(\alpha^2 - 6\alpha + 5)(\beta^2 - 6\beta + 18)}{\alpha^2 + \beta^2}$

$= \frac{(-1+5)(-1+18)}{34} = \frac{4}{2} = 2$ ，故選(A)

24. 滿意度 $= \frac{0.56 + 0.64}{2} = 0.6 = 60\%$
$\Rightarrow 1200 \times 60\% = 720$ ，故選(B)

25. 圖⑩有 $1 + 2 + 3 \cdots + 10 = \frac{(1+10)\times10}{2} = 55$ 個奇數
所有奇數數字之總和爲 $1 + 3 + 5 + 7 + 9 + \cdots + a_{55}$

$= \frac{55}{2}[2\times1 + (55-1)\times2] = \frac{55}{2}\times110 = 3025$ ，故選(D)